Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 26: Rules of Logarithms

Definition 1. **Product Rule:** *The logarithm of the product of two (or more) numbers is the sum of the logarithms of the numbers.*

$$\log_a(MN) = \log_a M + \log_a N.$$

For example, $\log_2(5.17) = \log_2 5 + \log_2 17$.

Definition 2. **Quotient Rule:** *The logarithm of the quotient of two numbers is the difference of the logarithms of the numbers.*

$$\log_a(\frac{M}{N}) = \log_a M - \log_a N.$$

For example, $\log_2(\frac{5}{17}) = \log_2 5 - \log_2 17$.

Definition 3. **Power Rule:** *The logarithm of a number to the power r is r times the logarithm of the number.*

$$\log_a(M)^r = r(\log_a M).$$

For example, $\log_2(17)^5 = 5(\log_2 17)$.

Example 1. *Given that* $\log_5 z = 3$ *and* $\log_5 y = 2$. *Find* $\log_5 y^3 z$.

Solution: *Using the rules*

$$\begin{aligned}\log_5 y^3 z &= \log_5 y^3 + \log_5 z \\ &= 3\log_5 y + \log_5 z \\ &= 3(2) + 3 \\ &= 9.\end{aligned}$$

Example 2. *Write the expression* $\ln(\frac{x^2(x-1)^3}{(2x+1)^4})$ *in expanded form.*

Solution: *Using the rules*

$$\begin{aligned}\ln(\frac{x^2(x-1)^3}{(2x+1)^4}) &= \ln x^2(x-1)^3 - \ln(2x+1)^4 \\ &= \ln x^2 + \ln(x-1)^3 - \ln(2x+1)^4 \\ &= 2\ln x + 3\ln(x-1) - 4\ln(2x+1).\end{aligned}$$

Example 3. *Write the expression* $\frac{1}{3}[\ln x + \ln(x+1) - \ln(x^2+1)]$ *in the condensed form (or as one logarithm).*

Solution: *Using the rules*

$$\begin{aligned}\frac{1}{3}[\ln x + \ln(x+1) - \ln(x^2+1)] &= \frac{1}{3}[\ln x(x+1) - \ln(x^2+1)] \\ &= \frac{1}{3}[\ln(\frac{x(x+1)}{x^2+1})] \\ &= \ln[\frac{x(x+1)}{x^2+1}]^{\frac{1}{3}} \\ &= \ln\sqrt[3]{\frac{x(x+1)}{x^2+1}}\end{aligned}$$

Definition 4. **Change-of-Base Formula:** *Let $a > 0$, $b > 0$ and $x > 0$ with $a \neq 1$ and $b \neq 1$. Then $\log_b x$ can be converted to a* ***different base*** *as follows:*

$$\log_b x = \frac{\log_a x}{\log_a b} = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}.$$